Anderson localization of electromagnetic waves in confined dielectric media

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Anderson localization of electromagnetic waves in random arrays of dielectric cylinders confined within a planar metallic waveguide is studied. The disordered dielectric medium is modeled by a system of randomly distributed two-dimensional electric dipoles. An effective theoretical approach based on the method of images is developed. A clear distinction between isolated localized waves (which exist already in finite media) and the band of localized waves (which appears only in the limit of the infinite medium) is presented. The Anderson localization emerging in the limit of an infinite medium is observed both in finite-size scaling analysis of transmission and in the properties of the spectra of some random matrices. [S1063-651X(99)04203-8]

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I. INTRODUCTION

The concept of Anderson localization originates from investigations of transport properties of electrons in noncrystalline systems such as amorphous semiconductors or disordered metals [1]. As pointed out by Anderson, in a sufficiently disordered solid an entire band of spatially localized electronic states can be formed [2,3]. It is natural to expect that the material in which an entire band of localized electronic states exists will be an insulator, whereas the case of extended states will correspond to a conductor [4]. In this way the phenomenon of Anderson localization may be referred to as a metal-insulator transition [5]. It is a common believe that localization is a pure interference effect and originates from multiple scattering of an electron from randomly distributed defects in the solid. On the other hand, the multiple scattering and interference are well known phenomena for electromagnetic waves and, consequently, several generalizations of electron localization to the realm of electromagnetic waves have been proposed [6–10].

The Anderson transition in the propagation of electromagnetic waves can best be observed experimentally in the scaling properties of the transmission T. Imagine a slab of thickness L containing randomly distributed nonabsorptive scatterers. Usually propagation of electromagnetic waves in weakly scattering random media can be described adequately by a diffusion process [11,12]. Thus the equivalent of Ohm's law holds and the transmission decreases linearly with the thickness of the sample, i.e., $T\propto L^{-1}$. However, when the fluctuations of the dielectric constant become large enough, the electromagnetic field ceases to diffuse and becomes localized due to interference. Anderson localization occurs when this happens. In such a case the material behaves as an optical equivalent of an insulator and the transmission decreases exponentially with the size of the system $T\propto e^{-L/\xi}$ [7,13].

A convincing experimental demonstration that Anderson transition is indeed possible in three-dimensional disordered dielectric structures has been given recently [14]. The strongly scattering medium has been provided by semiconductor powders with a very large refractive index. By decreasing the average particle size it was possible to observe a clear transition from linear scaling of transmission ($T\propto L^{-1}$) to an exponential decay ($T\propto e^{-L/\xi}$). Some localization effects have been also reported in previous experiments on microwave localization in copper tubes filled with metallic and dielectric spheres [15]. However, the latter experiments were plagued by large absorption, which makes the interpretation of the data quite complicated.

Another experiment on microwave localization has been performed in a two-dimensional medium [16]. The scattering chamber was set up as a collection of dielectric cylinders randomly placed between two parallel aluminum plates on half the sites of a square lattice. These authors attributed the observed sharp peaks of transmission to the existence of localized modes and measured the energy density of the electromagnetic field localized by their random structures.

In this paper we develop a simple yet reasonably realistic theoretical approach to Anderson localization of electromagnetic waves in two-dimensional dielectric media confined within a metallic waveguide. The results of our previous papers [17,18] dealing with the free space configurations are now extended to encompass the case of nontrivial boundary conditions. A sound physical interpretation in terms of transmission experiment is also proposed.

It should be stressed that the boundary conditions considered in this paper are different from those encountered in the experiment of Ref. [16]. To minimize the effect of the waves reflected off the edges of the scattering chamber, its perimeter was lined with a layer of microwave absorber. Therefore, to model that particular experiment, it is appropriate to use the free space boundary conditions (as we did in our previous papers [17,18]). Results presented here suggest, however, that it is easier to observe the localized states for dielectric cylinders confined to a metallic waveguide than in the case of open geometry. An experimental verification of our predictions seems feasible and it would allow a deeper understanding of localization phenomena in two-dimensional media.

By confining a system of randomly distributed dielectric cylinders into a planar metallic waveguide, we are able to observe clear signs of Anderson localization already for $N=100$ scatterers. One of the indicators of localization is the phase transition in the spectra of certain random matrices already discovered (for free space situation) in our previous paper [18]. This property of random Green functions is now generalized to the case of a confined dielectric medium (where the Green function is different). It may be interpreted as an appearance of the band of localized electromagnetic waves emerging in the limit of the infinite medium. A connection between this phenomenon and a dramatic inhibition of the propagation of electromagnetic waves in a spatially random dielectric medium is provided. A clear distinction between isolated localized waves (which do exist in finite confined media) and the band of localized waves (which appears in the limit of an infinite random medium) is also presented.

This paper is organized as follows. In Sec. II we introduce the basic assumptions of our model. The explicit expressions for the transmission and reflection coefficients are derived. In Sec. III the method of images is recalled and applied for our configuration. We arrive at the system of linear Lippmann-Schwinger-like equations determining the polarization of the medium for a given incident wave. In Sec. IV a numerical transmission experiment is performed. Sharp peaks of transmission are observed and attributed to the existence of localized modes. The finite size scaling of transmission indicates an exponential decay relevant for Anderson localization. In Sec. V localized waves are related to certain eigenvectors of the system of linear equations obtained in Sec. III. A phase transition in the spectra of random matrices whose elements are equal to the Green function calculated for the differences between positions of any pair of scatterers is discovered. It is interpreted as an appearance of a band of localized electromagnetic waves emerging in the limit of the infinite medium. We finish with a discussion as well as with some comments and conclusions in Sec. VI.

II. BASIC ASSUMPTIONS

In the following we study the properties of the stationary solutions of the Maxwell equations in two-dimensional media consisting of randomly placed parallel dielectric cylinders of infinite height (i.e., very long as compared to the wavelength of the electromagnetic field). This means that one, say (y), out of three dimensions is translationally invariant and only the remaining two (x,z) are random. The main advantage of this two-dimensional approximation is that we can restrict ourselves to the scalar theory of electromagnetic waves [17]:

$$\vec{E}(\vec{r},t)=\mathrm{Re}\{\vec{e}_y\,\mathcal{E}(x,z)\,e^{-i\omega t}\}. \tag{1}$$

Consequently, the polarization of the medium takes the form

$$\vec{P}(\vec{r},t)=\mathrm{Re}\{\vec{e}_y\,\mathcal{P}(x,z)\,e^{-i\omega t}\}. \tag{2}$$

The model predictions can be compared with rigorous numerical simulations [19–21].

Localization of electromagnetic waves in 2D media is usually studied experimentally in microstructures consisting of dielectric cylinders with diameters and mutual distances being comparable to the wavelength [16]. However, it seems to be a reasonable assumption that what really counts for the basic features of localization is the scattering cross section and not the real geometrical size of the scatterer itself. Therefore, we will represent the dielectric cylinders located at the points (x_a,z_a) by single 2D electric dipoles:

$$\mathcal{P}(x,z)=\sum_{a=1}^{N} p_a\,\delta^{(2)}(x-x_a,z-z_a). \tag{3}$$

In the present model we place the dielectric cylinders (3) between two infinite, perfectly conducting mirrors described by the equations $x=0$ and $x=d$. For simplicity, we consider only the case where the cylinders are oriented parallel to the mirrors. Moreover, our discussion will be restricted to the frequencies from the following range:

$$\pi<k\,d<2\pi, \tag{4}$$

where $k=\omega/c$ is the wave number in vacuum. Thus in the planar waveguide formed by the two parallel mirrors separated by a distance d, only *one* guided TE mode exists [22]:

$$\mathcal{E}^{(0)}(x,z)=\frac{2}{\sqrt{\beta\,d}}\sin(\alpha\,x)\,e^{i\beta z}, \tag{5}$$

where the propagation constants are given by

$$\alpha=\frac{\pi}{d},\qquad \beta=\sqrt{k^2-\alpha^2}. \tag{6}$$

A brief discussion of other cases is given in the final section of this paper.

The total field that can be measured *far* from the cylinders is fully described by the reflection ρ and transmission τ coefficients:

$$\mathcal{E}(x,z)=\begin{cases}\mathcal{E}^{(0)}(x,z)+\rho\,\mathcal{E}^{(0)*}(x,z) & \text{for } z\to-\infty\\ \tau\,\mathcal{E}^{(0)}(x,z) & \text{for } z\to+\infty.\end{cases} \tag{7}$$

Using the Lorentz theorem and repeating the straightforward but lengthly calculations (see, e.g., [22]) we easily arrive at the following expressions determining the transmission coefficient:

$$\tau=1+i\pi k^2\sum_{a=1}^{N} p_a\,\mathcal{E}^{(0)*}(x_a,z_a) \tag{8}$$

and the reflection coefficient

$$\rho=i\pi k^2\sum_{a=1}^{N} p_a\,\mathcal{E}^{(0)}(x_a,z_a), \tag{9}$$

for given dipole moments p_a. In the following section we will relate p_a to the values of the incident field calculated at the positions of the cylinders $\mathcal{E}^{(0)}(x_a,z_a)$.

III. METHOD OF IMAGES

A simple way to take into account the boundary conditions of parallel mirrors and their influence on the electromagnetic field is to use the method of images. This technique has been used, i.e., in QED calculations of spontaneous emission in cavities [23,24]. To reproduce the correct boundary conditions on the radiation field of each cylinder (3), the mirrors are replaced by an array of image cylinders whose phases alternate in sign:

$$\mathcal{P}(x,z)=\sum_{a=1}^{N}\sum_{j=-\infty}^{\infty}(-1)^{j}p_{a}\,\delta^{(2)}(x-x_{a}^{(j)},z-z_{a}),\quad(10)$$

where

$$x_{a}^{(j)}=(-1)^{j}x_{a}+jd.\quad(11)$$

Thus a finite system of dielectric cylinders (3) placed within a metallic waveguide is fully equivalent to an infinite system of cylinders (10) forming a slab in a free space. This fact allows us to utilize some results from our previous paper concerning dielectric cylinders in free space [17].

It is now a well-established fact that to use safely the point-scatterer approximation it is essential to use a representation for the cylinders that conserves energy in the scattering processes. In the case of a system of cylinders in free space, this requirement gives the following form of the coupling between the dipole moment p_a and the electric field incident on the cylinder $\mathcal{E}'(x_a,z_a)$ [17]:

$$i\pi k^{2}p_{a}=\frac{e^{i\phi}-1}{2}\mathcal{E}'(x_{a},z_{a}).\quad(12)$$

The same result holds also for a system of cylinders placed in a metallic waveguide (10). In this case the field acting on the ath cylinder,

$$\mathcal{E}'(x_{a},z_{a})=\mathcal{E}^{(0)}(x_{a},z_{a})+\frac{e^{i\phi}-1}{2}\sum_{b=1}^{N}G_{ab}\,\mathcal{E}'(x_{b},z_{b}),\quad(13)$$

is the sum of the incident guided mode $\mathcal{E}^{(0)}$, which obeys the Maxwell equations in an empty waveguide, and waves scattered by all other cylinders *and* by all images. Thus, in the present model the G matrix from Eq. (13) needs to be defined differently than in Ref. [17]:

$$i\pi G_{ab}=2\sum_{\rho_{ab}^{(j)}\neq 0}(-1)^{j}K_{0}(-ik\rho_{ab}^{(j)}),\quad(14)$$

where

$$\rho_{ab}^{(j)}=\sqrt{(x_{a}-x_{a}^{(j)})^{2}+(z_{a}-z_{b})^{2}}\quad(15)$$

denotes the distance between the ath cylinder and the jth image of the bth cylinder and K_0 is the modified Bessel function of the second kind. Note that summation in Eq. (14) is performed over all j, for which $\rho_{ab}^{(j)}\neq 0$.

The system of linear equations (13) fully determines the field acting on each cylinder $\mathcal{E}'(x_a,z_a)$ for a given field of the guided mode $\mathcal{E}^{(0)}(x_a,z_a)$ incident on the system. Analogous relationships between the stationary outgoing wave and the stationary incoming wave are known in the general scattering theory as the Lippmann-Schwinger equations [25]. If we solve Eqs. (13) and use Eqs. (12) to find p_a, then we are able to find the transmission and reflection coefficients given by Eqs. (8) and (9).

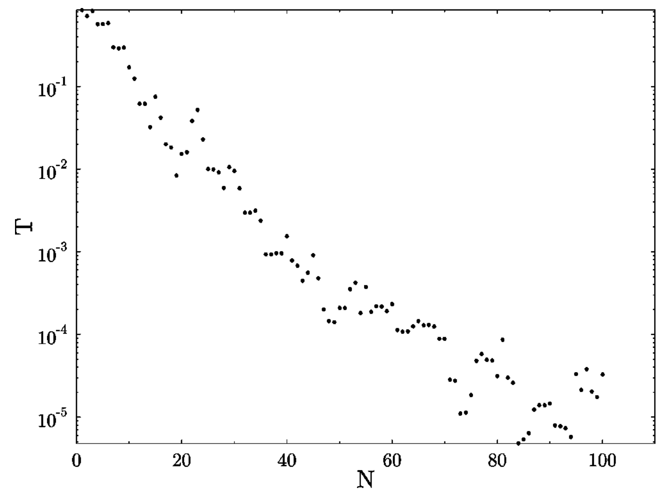

FIG. 1. Transmission T of the system of dielectric cylinders described by the phase shifts $\phi=-1$ placed randomly in a planar metallic waveguide plotted as a function of the number of cylinders N.

IV. TRANSMISSION EXPERIMENT

The actual properties of physical systems have to be observed experimentally and it is not enough just to know the properties of the stationary solutions of the Maxwell equations. These are only theoretical tools. Experiments deal rather with measurable quantities and, for many practical problems, a natural quantity to look for is the transmission $T=|\tau|^2$ of a finite system of characteristic size L and its dependence on L. As a simple example let us consider a system of N cylinders placed between the mirrors separated by a distance $kd=3\pi/2$. The cylinders are distributed randomly with constant uniform density $n=1$ cylinder per wavelength squared. Therefore for each N the size of the system is proportional to the number of cylinders $L\propto N$. The images of the cylinders from Eq. (10) were summed from $j=-50\,000$ to $j=50\,000$. In Fig. 1 we present the log-linear plot of the transmission T as a function of the number of cylinders N calculated for a fixed value of $\phi=-1$. It follows from inspection of this figure that $T\propto e^{-L/\xi}$. This proves that in the limit $L\rightarrow\infty$ our system indeed supports a localized state and behaves as an optical insulator. Consequently, Anderson localization occurs when this happens.

In Fig. 2 we plot the transmission T of the systems of $N=1,10,100$ cylinders as a function of the phase shift of a single cylinder ϕ. We see that in the case $N=1$ the incident wave is totally reflected for a *single* value of $\phi=\phi_0$. Note that not necessarily $|\phi_0|=\pi$, and therefore for this value of ϕ the total scattering cross section σ of an individual dielectric cylinder [17,18],

$$k\sigma=2(1-\cos\phi),\quad(16)$$

does not approach its maximal value. However, for systems containing $N=10$ and $N=100$ the entire *regions* of the values of phase shifts ϕ exist for which $T\simeq 0$. They are separated by narrow maxima of transmission. Moreover, inspection of Fig. 2 suggests that in the limit $N\rightarrow\infty$ the number of these maxima increases and simultaneously they became narrower and sharper. Therefore, we may expect that for suffi-

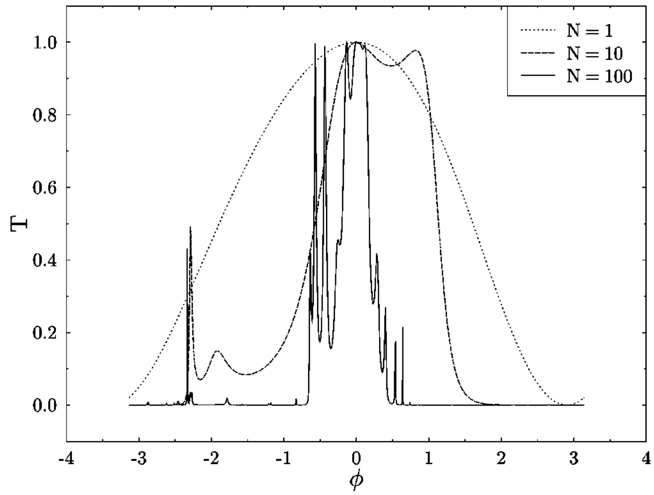

FIG. 2. Transmission T of the systems of $N=1,10,100$ dielectric cylinders placed randomly in a planar metallic waveguide plotted as a function of the phase shift of a single cylinder ϕ.

ciently large N the incident waves will be totally reflected for almost any ϕ except the discrete set $\phi=\phi_l$ for which the transmission is close to unity. Physically speaking this means that different realizations of a sufficiently large system of randomly placed cylinders are hardly distinguishable from each other by a transmission experiment.

It follows from inspection of Eqs. (8) and (9) that the maximum of transmission $T=|\tau|^2=1$ (and minimum of reflection $R=|\rho|^2=0$, because the medium is nondissipative) corresponds to the case when the polarization of the medium fulfills the following condition:

$$\sum_{a=1}^{N} p_a \mathcal{E}^{(0)*}(x_a,z_a)=\sum_{a=1}^{N} p_a \mathcal{E}^{(0)}(x_a,z_a)=0. \tag{17}$$

This means that in the expansion of the field radiated by the medium into waveguide modes, the coefficients near guided modes Eq. (5) vanish. Therefore the radiated field consists only of evanescent modes with imaginary propagation constants β and thus it is exponentially localized in the vicinity of the medium. In the next section we will show that such a field can exist also without any incident wave and therefore represents a truly *localized* wave.

Analogously, for a random and infinite *one-dimensional* system one can prove mathematically [26] that incident waves are totally reflected for ''almost any'' frequency, i.e., except the discrete set (of zero measure) for which the transmission is equal to unity. It can be shown that this dense set of discrete frequencies (see the Furstenberg theorem [27]) corresponds to the band of localized one-dimensional waves.

Let us mention that according to the scaling theory of localization [13], any degree of disorder will lead to localization in one and two dimensions, while in three dimensions a certain critical degree of disorder is needed before localization will set in. Our calculations certainly do not exclude the possibility that in an infinite 2D medium Anderson localization may occur for $\phi\to 0$. However, as follows from Fig. 2 (dealing with finite media), with increasing size of the system the signs of Anderson localization appear *faster* for $|\phi|\simeq\pi$ than for other values of ϕ.

V. LOCALIZED WAVES

By definition, an electromagnetic wave is localized in a certain region of space if its magnitude is (at least) exponentially decaying in any direction from this region. We will show now that electromagnetic waves localized in the system of dielectric cylinders placed in a planar metallic waveguide correspond to nonzero solutions $\mathcal{E}_l'(x_a,z_a)\neq 0$ of Eqs. (13) for the incoming wave equal to zero, i.e., $\mathcal{E}^{(0)}(x,z)\equiv 0$. Note that we added an index l which labels the localized waves.

Indeed, let us suppose that the field is exponentially localized. This means that there are no guided modes in the radiation field. Therefore (as shown in the preceding section), Eq. (17) holds. Using Eq. (12), we see that the vector formed by the values of the field acting on the cylinders is *orthogonal* to the vector formed by the values of incident field calculated at the positions of the cylinders:

$$\sum_{a=1}^{N} \mathcal{E}_l'(x_a,z_a)\,\mathcal{E}^{(0)*}(x_a,z_a)=0. \tag{18}$$

But simultaneously $\mathcal{E}_l'(x_a,z_a)$ is a solution of a system of linear Eqs. (13) where $\mathcal{E}^{(0)}(x_a,z_a)$ is the right-hand side. Therefore $\mathcal{E}_l'(x_a,z_a)$ is also a solution of Eqs. (13) with $\mathcal{E}^{(0)}(x_a,z_a)\equiv 0$. Note that in this case the latter system of equations is equivalent to the *eigenproblem* for the G_{ab} matrix:

$$\sum_{b=1}^{N} G_{ab}\,\mathcal{E}_l'(x_b,z_b)=\lambda_l\,\mathcal{E}_l'(x_a,z_a), \tag{19}$$

where

$$\frac{1}{\lambda_l}=\frac{e^{i\phi_l}-1}{2}. \tag{20}$$

The proof works also the other way round. Suppose that $\mathcal{E}_l'(x_a,z_a)$ is a solution of Eqs. (13) for $\mathcal{E}^{(0)}(x_a,z_a)\equiv 0$. As the considered medium is nondissipative, the time average energy stream integrated over a closed surface surrounding it must vanish. This means that there are again no guided modes in the radiation field [which in the case $\mathcal{E}^{(0)}(x,z)\equiv 0$ is equal to the total field]. Therefore Eq. (17) holds and the wave is localized.

Let us stress that Eq. (20) can be fulfilled *only* if the real part of an eigenvalue satisfies

$$\mathrm{Re}\,\lambda_l=-1. \tag{21}$$

The imaginary part of the eigenvalue and the phase shift are then related by

$$\tan\frac{\phi_l}{2}=-\frac{1}{\mathrm{Im}\,\lambda_l}. \tag{22}$$

Therefore only those eigenvectors $\mathcal{E}_l'(x_a,z_a)$ of the G_{ab} matrix which correspond to the eigenvalues λ_l satisfying the condition (21) may be related to localized waves. Moreover, those waves can exist only if the phase shift ϕ which determines the scattering properties of the cylinders is equal to the

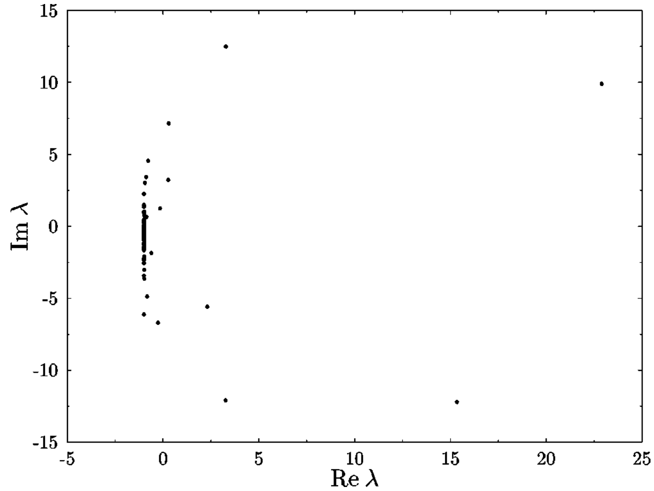

FIG. 3. Spectrum λ of the matrix G_{ab} corresponding to a system of $N=100$ dielectric cylinders placed randomly in a planar metallic waveguide.

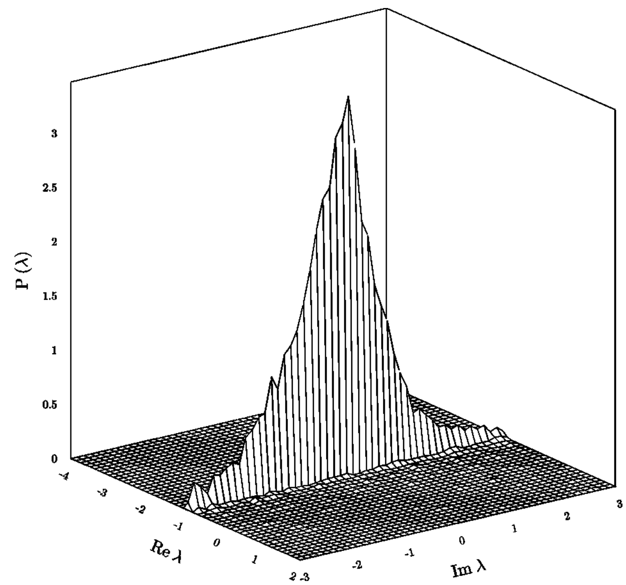

FIG. 4. Density of eigenvalues $P(\lambda)$ of the matrix G_{ab} calculated from 10^2 distributions of $N=100$ cylinders.

corresponding value ϕ_l given by Eq. (22). As discussed in Sec. IV, they are the same values of ϕ for which the transmission is equal to unity.

Let us now apply our model to a certain system of cylinders located at fixed points (x_a, z_a) between the mirrors separated by a fixed distance d. These cylinders are described by a dielectric constant $\epsilon(\omega)$ and radius R. Thus the parameter ϕ from Eq. (12) which is directly related to the total scattering cross section σ of an individual dielectric cylinder still remains a function of the frequency, i.e., $\phi=\phi(\omega)$. Note that the G_{ab} matrix from Eq. (14) (and its eigenvalues λ_l) depends on positions of the cylinders $(k\,x_a\,,\,k\,z_a)$ and the thickness of the waveguide $k\,d$ both rescaled in wavelengths. Thus the values of ϕ_l given by Eq. (22) which correspond to localized waves should also be regarded as functions of frequency, i.e., $\phi_l=\phi_l(\omega)$. This means that in the system considered, the localized waves (and resonances of transmission) appear at *discrete* frequencies ω_l determined by the crossing points of the functions ϕ and ϕ_l:

$$\phi(\omega_l)=\phi_l(\omega_l). \tag{23}$$

Note that in finite dielectric media no localized states are supported by Maxwell's equations in two dimensions [17]. However, this is not the case with confined media, where localized waves do exist even in finite media. Therefore, a clear distinction between localized waves with isolated frequencies and the dense band of localized waves (due to Anderson localization) is needed. We show now that this distinction may be provided by investigation of a phase transition which occurs in the limit of $N\to\infty$ in the spectra of G_{ab} matrices corresponding to systems of randomly distributed dielectric cylinders.

To support this statement in Fig. 3 we plot the spectrum λ_l of a G matrix (diagonalized numerically) corresponding to a certain specific configuration of $N=100$ cylinders placed randomly with the uniform density $n=1$ cylinder per wavelength squared. We see that quite a lot of eigenvalues are located near the $\mathrm{Re}\,\lambda=-1$ axis. As will be discussed below, this is a universal property of 2D G matrices, not restricted to this specific realization of the system only. To prove these statements we diagonalize numerically the G matrix (14) for 10^2 different distributions of $N=100$ cylinders. Then we construct a two-dimensional histogram of eigenvalues λ_l from all distributions. It approximates the corresponding probability distribution $P(\lambda)$ which is normalized in the standard way $\int d^2\lambda\, P(\lambda)=1$. In Fig. 4 we have the surface plot of the function $P(\lambda)$. It clearly shows that for all configurations (without, maybe, a set of zero measure) most eigenvalues are located near the $\mathrm{Re}\,\lambda=-1$ axis. This tendency is more and more pronounced with increasing size of the system measured by N. Our numerical investigations indicate that in the limit of an infinite medium, the probability distribution under consideration will tend to the δ function in the real part:

$$\lim_{N\to\infty} P(\lambda)=\delta(\mathrm{Re}\,\lambda+1)\, f(\mathrm{Im}\,\lambda). \tag{24}$$

This means that in this limit for almost any random distribution of the cylinders, an infinite number of eigenvalues satisfies the condition Eq. (21). It is therefore reasonable to expect that in the case of a random and infinite system a countable set of frequencies ω_l corresponding to localized waves becomes *dense* in some finite interval. But it is always difficult to separate such frequencies from frequencies which may be arbitrarily near and physically the spectrum is always a coarse-grained object. Therefore in the limit of an infinite medium an entire *band* of spatially localized electromagnetic waves appears.

VI. CONCLUDING REMARKS

Anderson localization of electromagnetic waves in random arrays of N dielectric cylinders confined within a planar metallic waveguide of thickness d has been studied. In studying the properties of the stationary solutions of the Maxwell equations in such two-dimensional media, several particular cases may be considered ($k\,d=\pi$ is the cutoff thickness of the waveguide). For $N=0$ and $k\,d<\pi$ there are no guided modes in the waveguide as well as no localized waves. This case is analogous to the electronic band gap in a solid. If $N>0$ and $k\,d<\pi$ there are still no guided modes in the waveguide but localized waves can appear for any distribution of the cylinders. It is again analogous to the solid state physics situation where isolated perturbations of the periodicity of crystals (like impurities or lattice defects) can lead to the formation of localized electronic states with energies within the forbidden band. Another possibility corresponds to $N=0$ and $k\,d>\pi$. In this case there are guided modes but the system supports no localized waves. This is very similar to the conductance band in solids. Guided modes correspond to extended electronic states described by Bloch functions. In this paper we have performed a detailed study of the regime where $N>0$ and $k\,d>\pi$. For this range of parameters there are both guided modes and resonances of transmission. Isolated localized waves can be seen for certain distributions of the cylinders. The signs of Anderson localization emerging in the limit of an infinite medium can be observed both in analysis of transmission and in the properties of the spectra of certain random matrices. Eventually let us consider a limiting case of $N\to\infty$ and $k\,d>\pi$. Now the guided modes no longer exist in the waveguide. Instead a band of localized waves will be formed for any distribution of the cylinders. It is an interesting analog of the Anderson localization in noncrystalline solids such as amorphous semiconductors or disordered metals.

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[1] N. F. Mott and E. A. Davis, *Electronic Processes in Non-Crystalline Materials* (Oxford University Press, New York, 1979).
[2] P. W. Anderson, Phys. Rev. **109**, 1492 (1958).
[3] P. W. Anderson, Rev. Mod. Phys. **50**, 191 (1978).
[4] B. Souillard, in *Chance and Matter*, edited by J. Souletie, J. Vannimenus, and R. Stora (North-Holland, Amsterdam, 1987), p. 305.
[5] N. F. Mott, *Metal-Insulator Transitions* (Taylor and Francis, London, 1974).
[6] S. John, Phys. Rev. Lett. **53**, 2169 (1984).
[7] P. W. Anderson, Philos. Mag. B **52**, 505 (1985).
[8] S. John, Phys. Rev. Lett. **58**, 2486 (1987).
[9] M. P. van Albada, A. Lagendijk, and M. B. van der Mark, in *Analogies in Optics and Micro Electronics*, edited by W. van Haeringen and D. Lenstra (Kluwer, Dordrecht, 1990), p. 85.
[10] M. Kaveh, in *Analogies in Optics and Micro Electronics*, edited by W. van Haeringen and D. Lenstra (Kluwer, Dordrecht, 1990), p. 21.
[11] A. Ishimaru, *Wave Propagaton and Scattering in Random Media* (Academic, New York, 1978).
[12] A. Kerker, *The Scattering of Light and Other Electromagnetic Radiation* (Academic, New York, 1969).
[13] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
[14] D. S. Wiersma, P. Bartolini, A. Lagendijk, and R. Righini, Nature (London) **390**, 671 (1997).
[15] A. Z. Genack and N. Garcia, Phys. Rev. Lett. **66**, 2064 (1991).
[16] R. Dalichaouch, J. P. Armstrong, S. Schultz, P. M. Platzman, and S. L. McCall, Nature (London) **354**, 53 (1991).
[17] M. Rusek and A. Orłowski, Phys. Rev. E **51**, R2763 (1995).
[18] M. Rusek, A. Orłowski, and J. Mostowski, Phys. Rev. E **56**, 4892 (1997).
[19] D. Felbacq, G. Tayeb, and D. Maystre, J. Opt. Soc. Am. A **11**, 2526 (1994).
[20] D. Maystre, Pure Appl. Opt. **3**, 975 (1994).
[21] D. Felbacq, D. Maystre, and G. Tayeb, J. Mod. Opt. **42**, 473 (1995).
[22] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962).
[23] J. P. Dowling, M. O. Scully, and F. De Martini, Opt. Commun. **82**, 415 (1991).
[24] J. P. Dowling, Found. Phys. **23**, 895 (1993).
[25] B. A. Lippmann and J. Schwinger, Phys. Rev. **79**, 469 (1950).
[26] F. Deylon, H. Kunz, and B. Souillard, J. Phys. A **16**, 25 (1983).
[27] H. Furstenberg, Trans. Am. Math. Soc. **108**, 377 (1963).